



Aalto University
School of Science

Temporal motifs in time-dependent networks

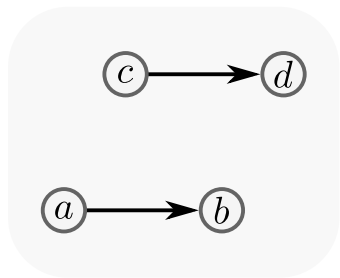
Lauri Kovanen

Department of Biomedical Engineering
and Computational Science

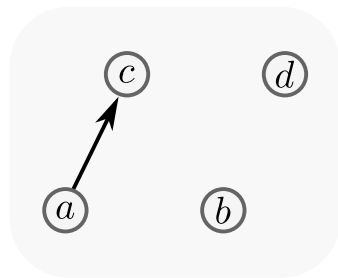
October 18th 2011

Finland, 1989

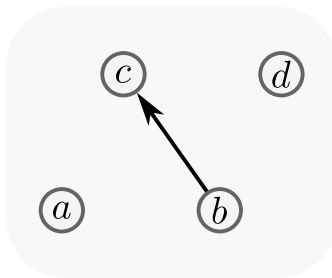




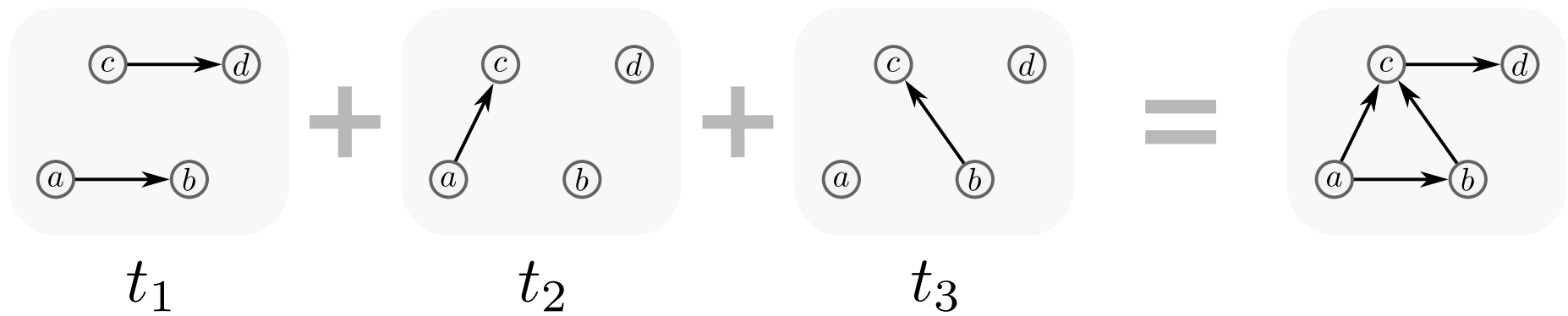
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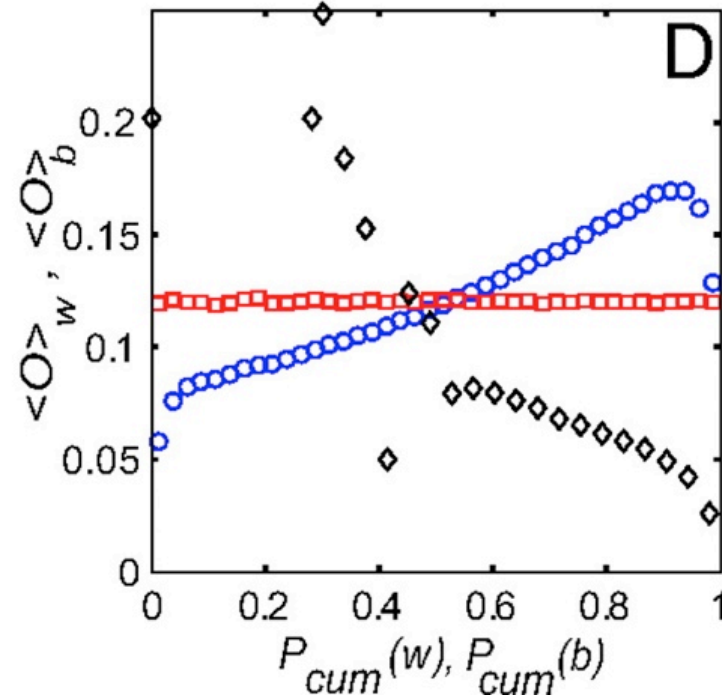
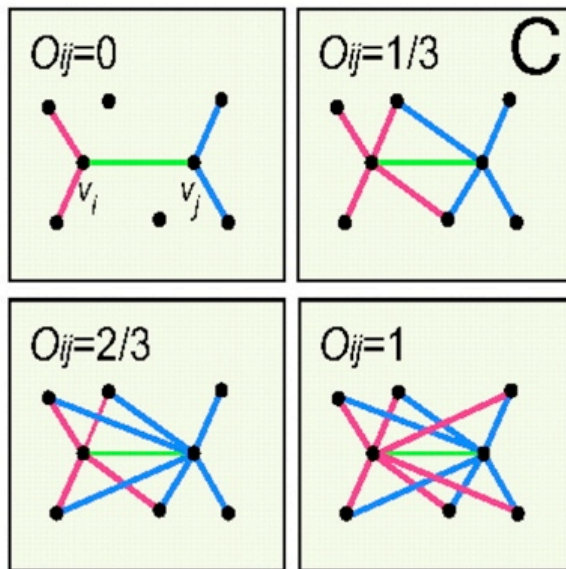


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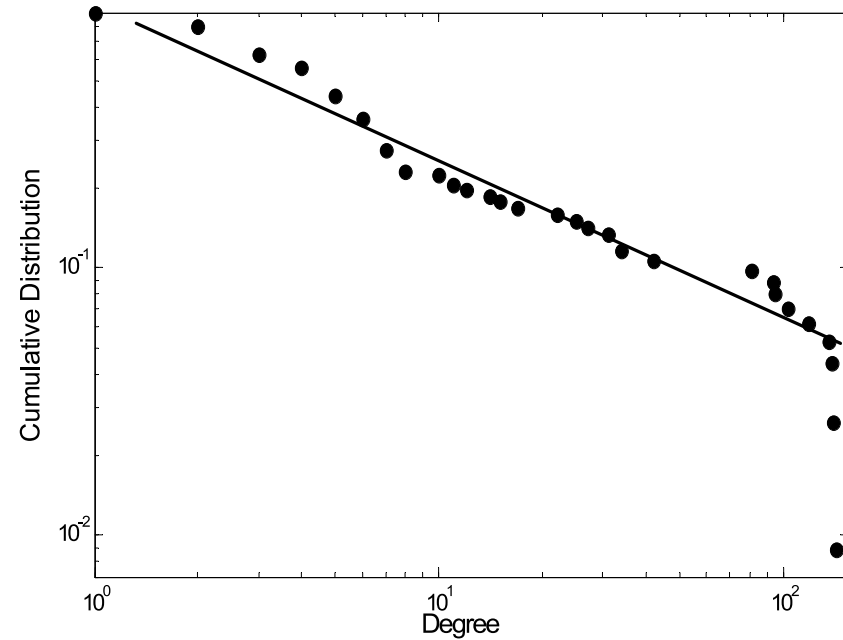
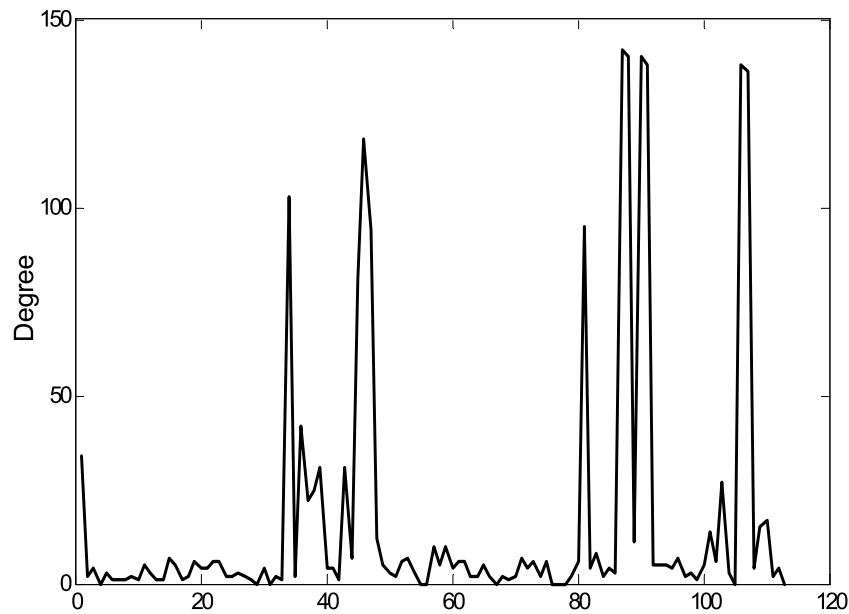


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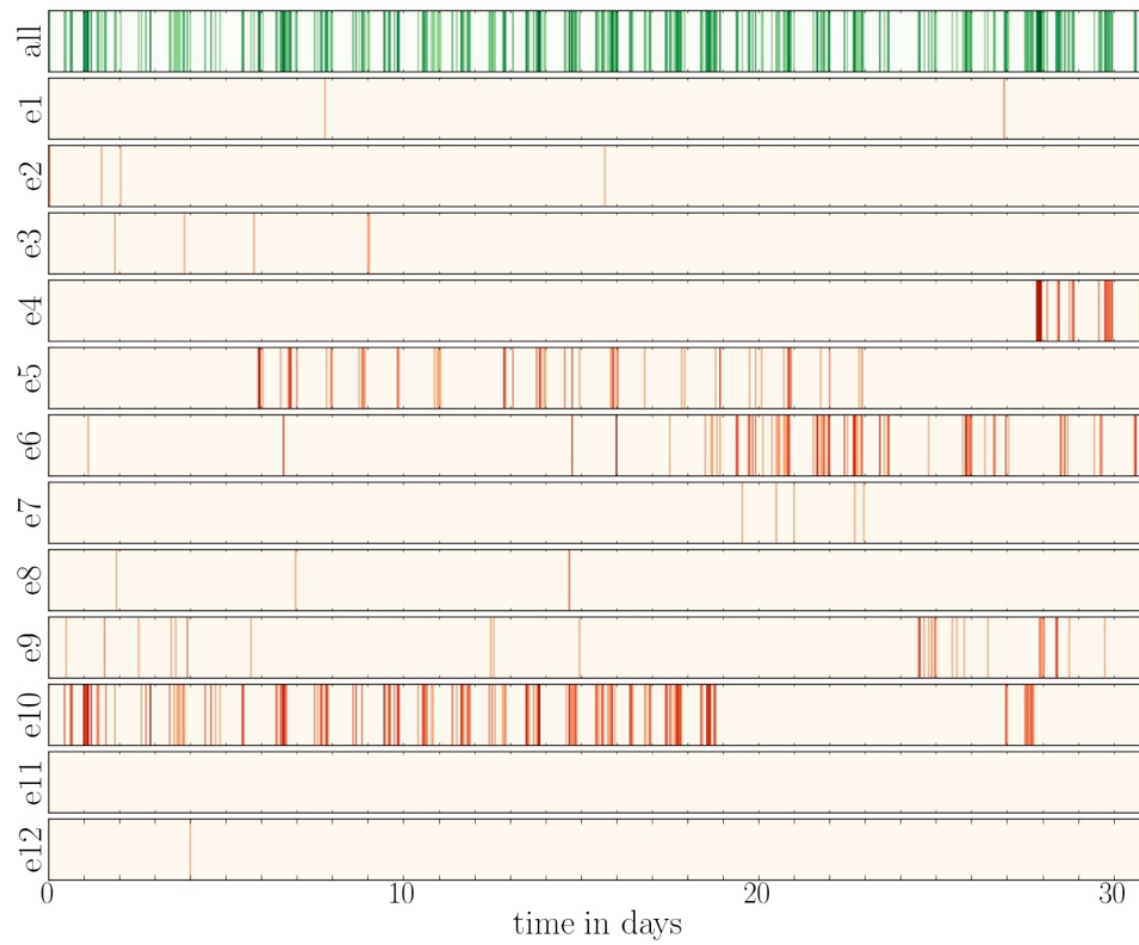


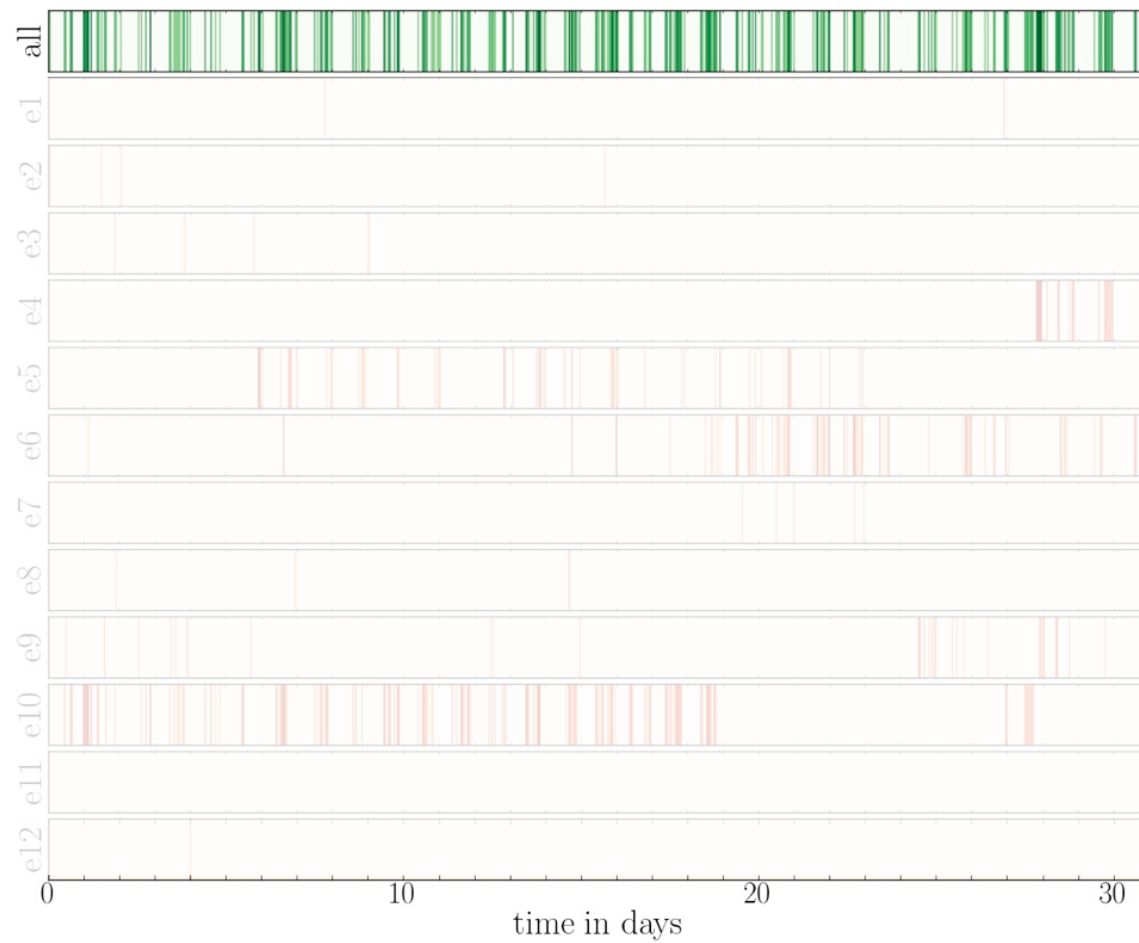


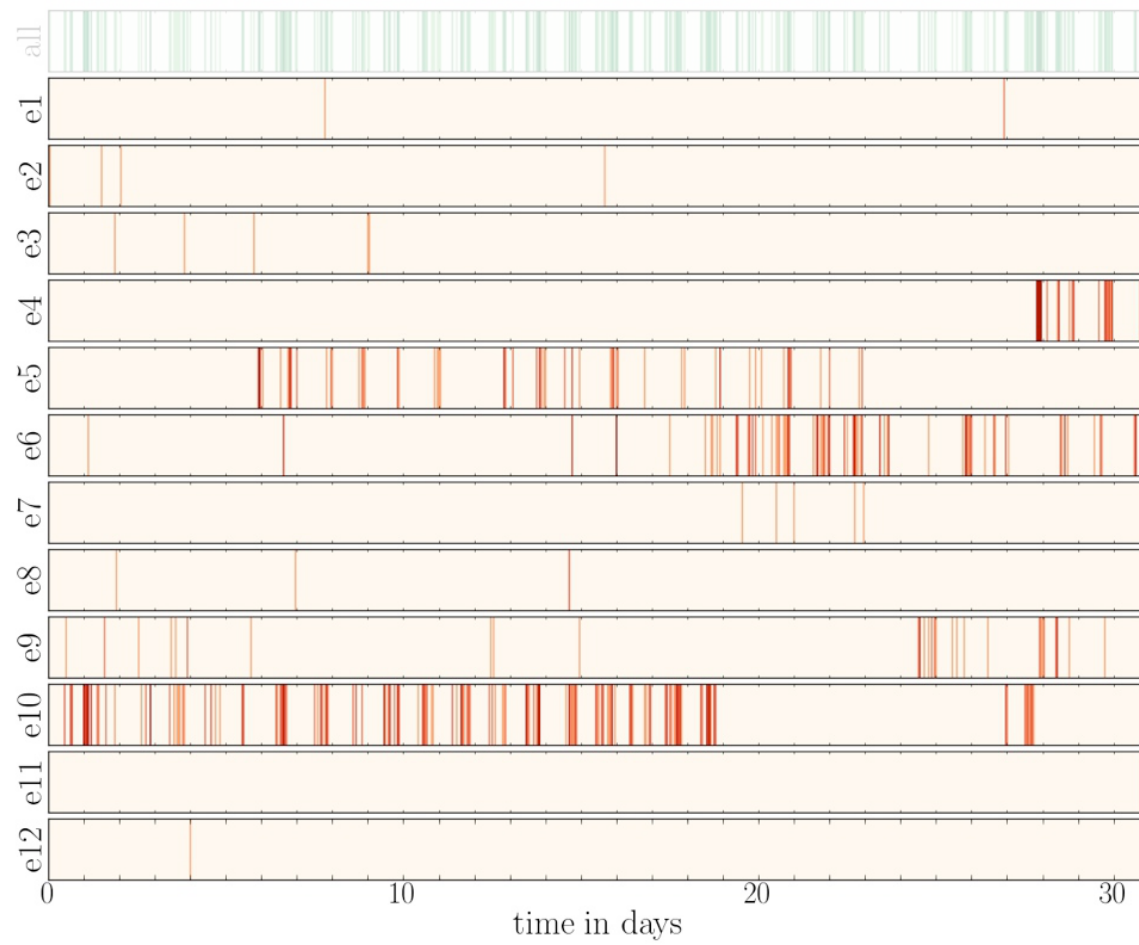
Onnela, J.-P. *et al.* Structure and tie strengths in mobile communication networks. *Proceedings of the National Academy of Sciences* **104**, 7332-7336 (2007).

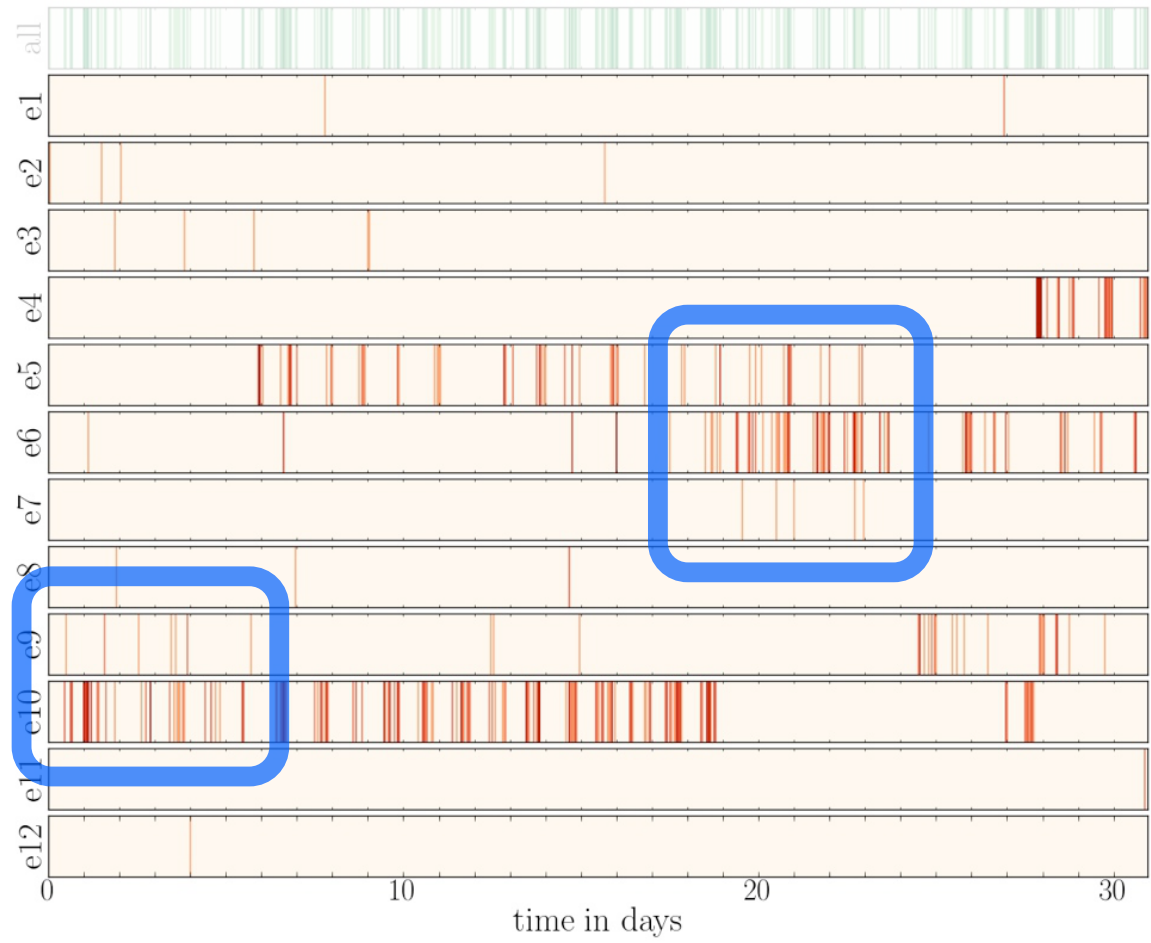


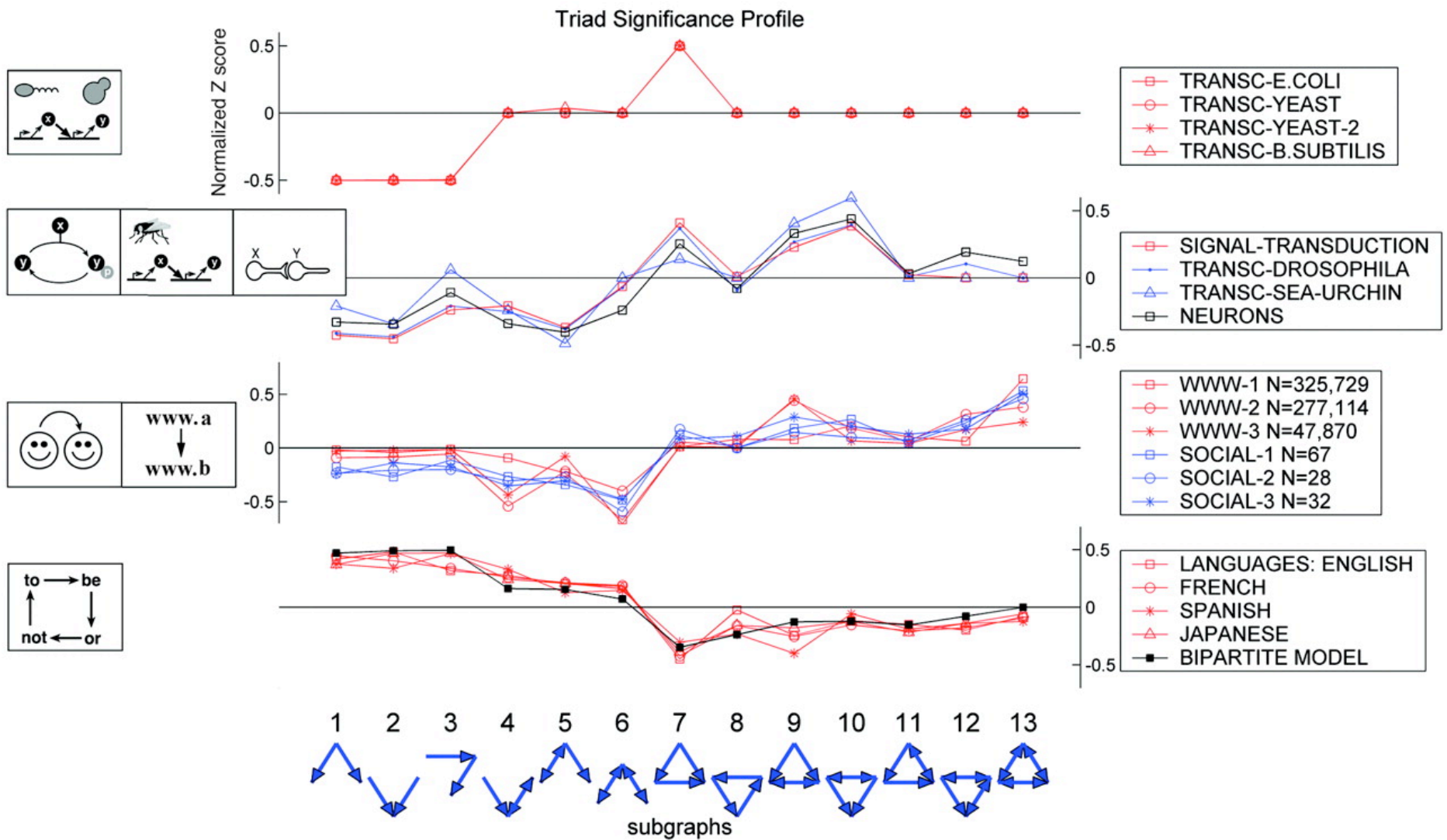
Braha, D. & Bar-Yam, Y. *Time-Dependent Complex Networks: Dynamic Centrality, Dynamic Motifs, and Cycles of Social Interaction*, 39-50. Springer Studies on Complexity (Springer Berlin / Heidelberg, 2009).









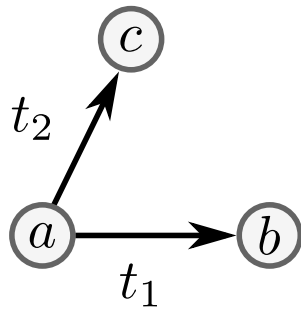


Milo, R. *et al.* Superfamilies of evolved and designed networks. *Science* **303**, 1538-1542 (2004).

Motif: Equivalence class of connected subgraphs.

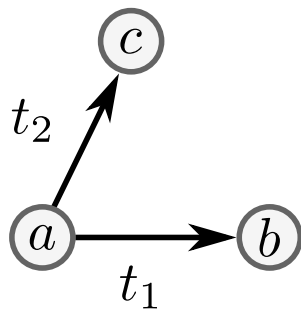
Motif: Equivalence class of *connected subgraphs*.

Δt - adjacency

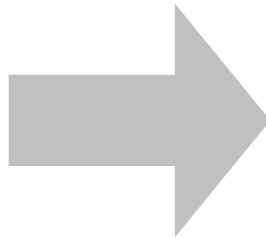


$$t_2 - t_1 \leq \Delta t$$

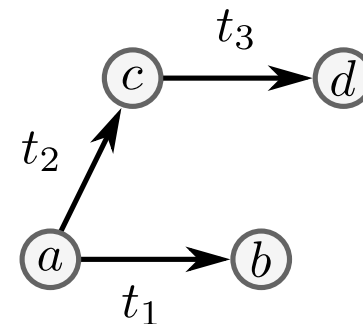
Δt - adjacency



$$t_2 - t_1 \leq \Delta t$$



Δt - connectivity

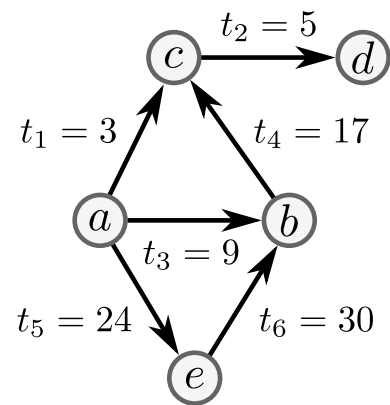


$$t_2 - t_1 \leq \Delta t \text{ and } t_3 - t_2 \leq \Delta t$$

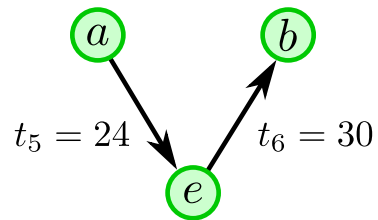
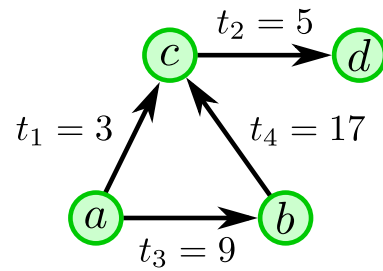
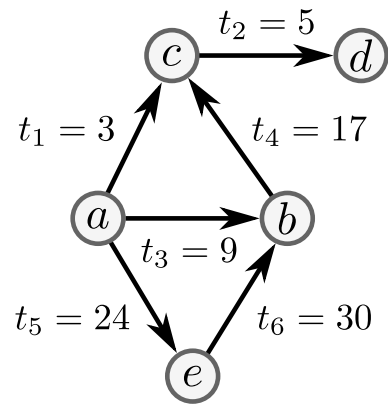
Temporal subgraph:

Sets of events where all event pairs are Δt - connected

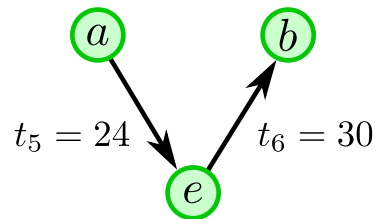
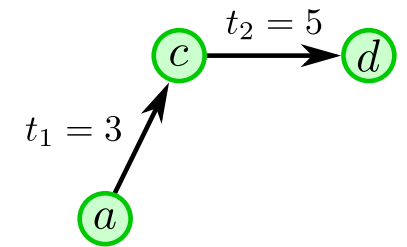
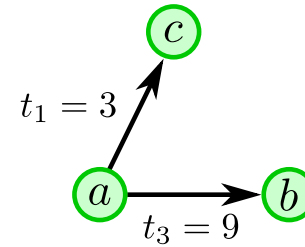
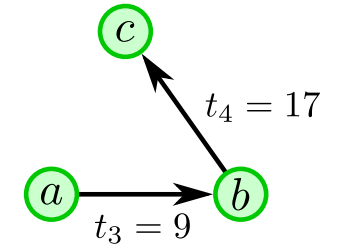
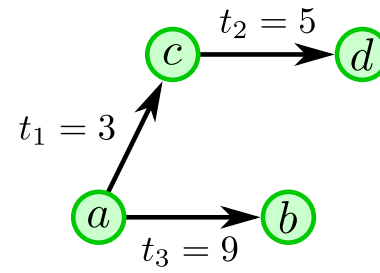
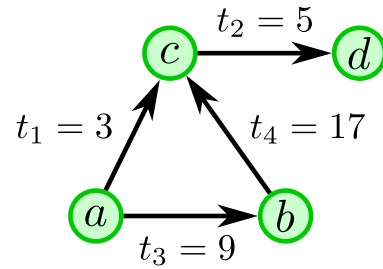
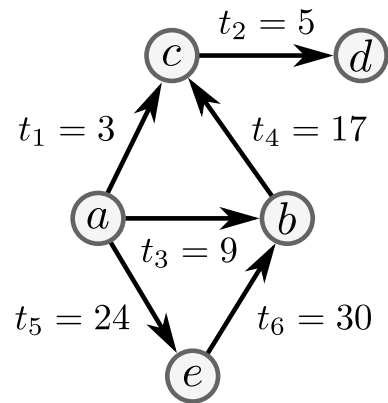
$$\Delta t = 10$$



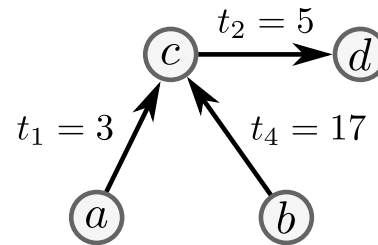
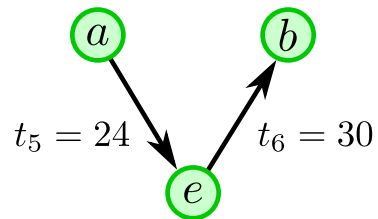
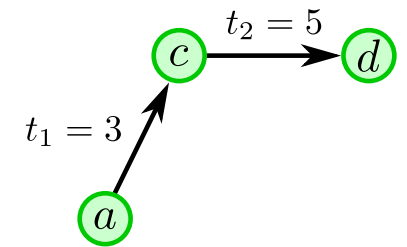
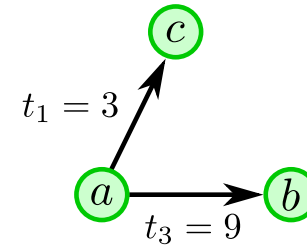
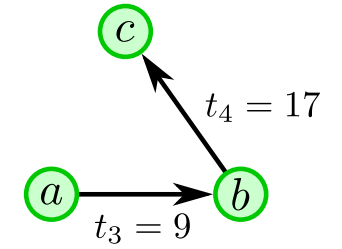
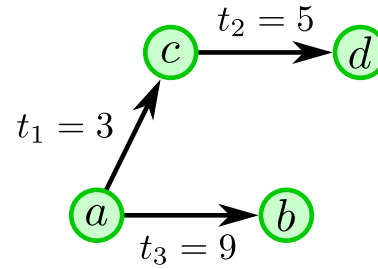
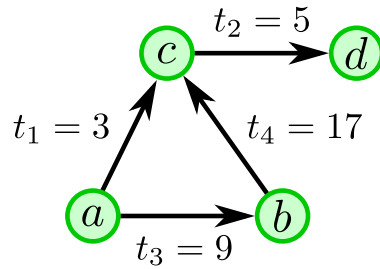
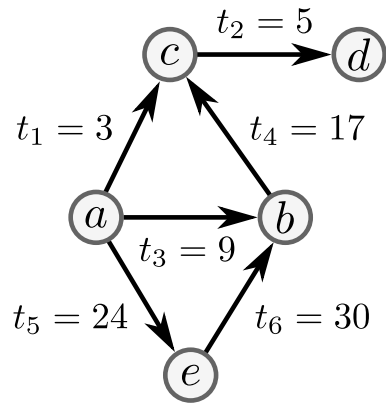
$$\Delta t = 10$$



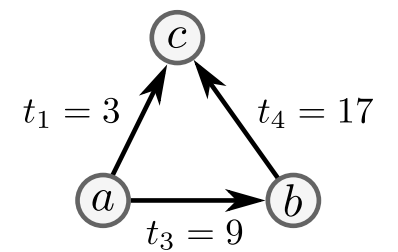
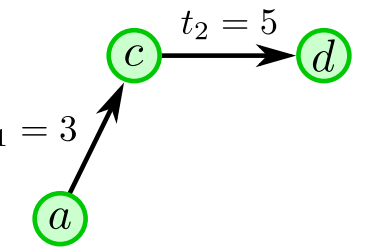
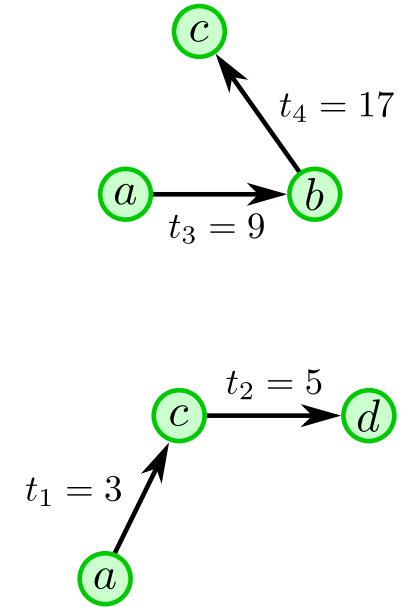
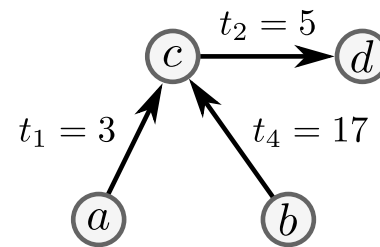
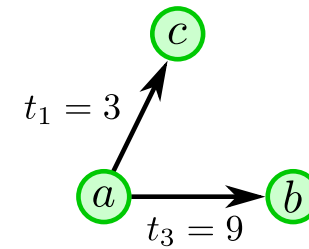
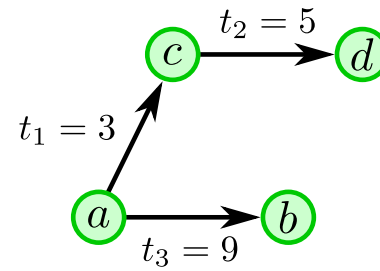
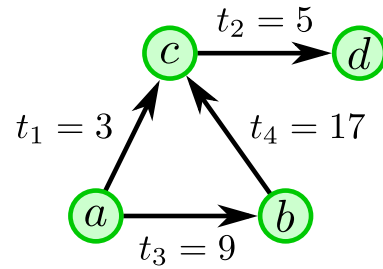
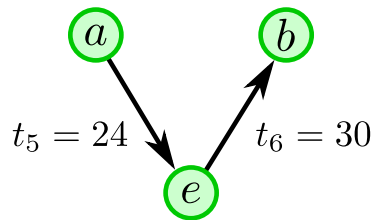
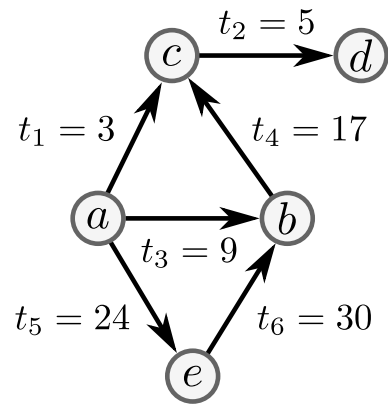
$\Delta t = 10$

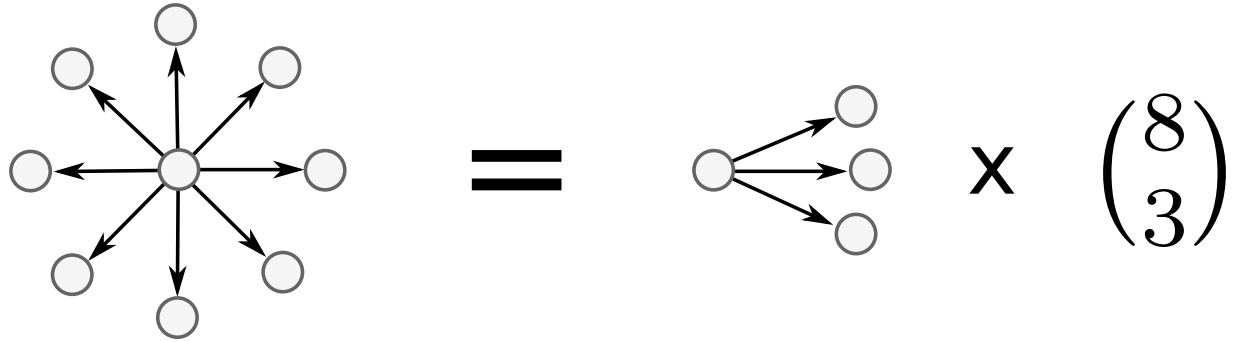


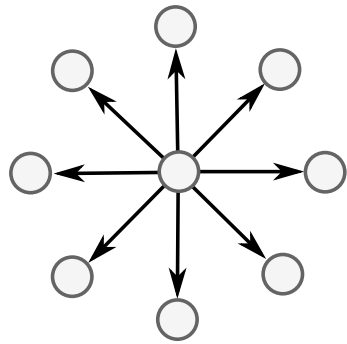
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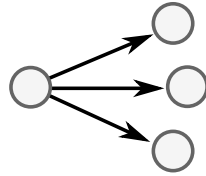
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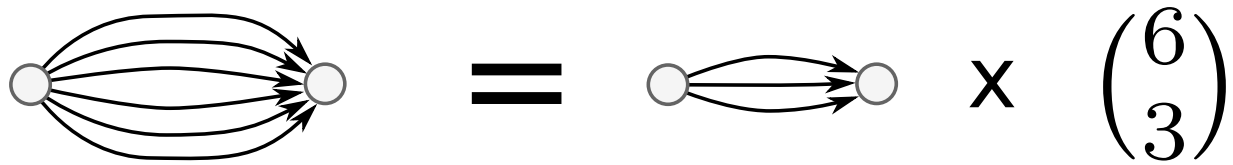
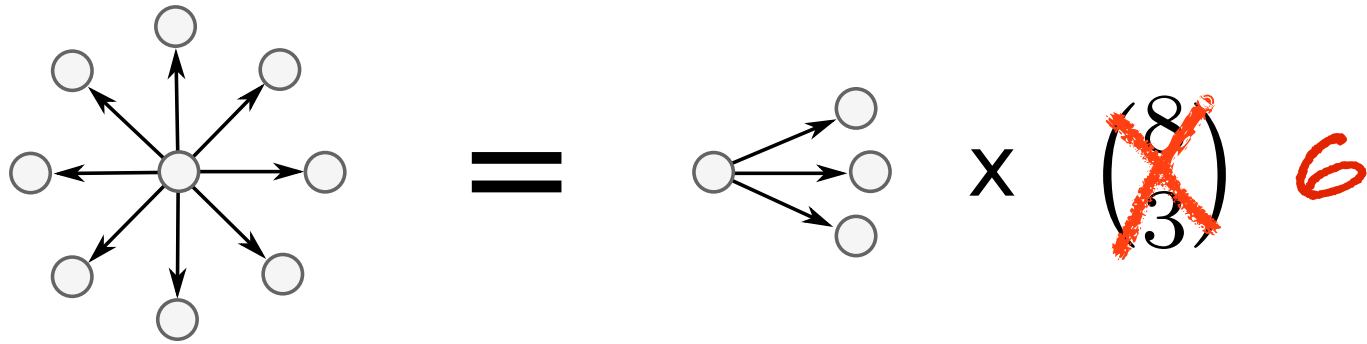
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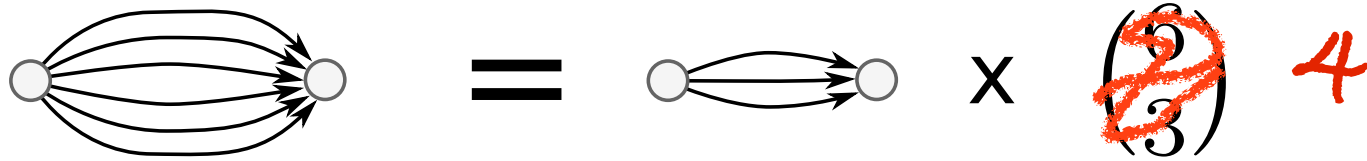
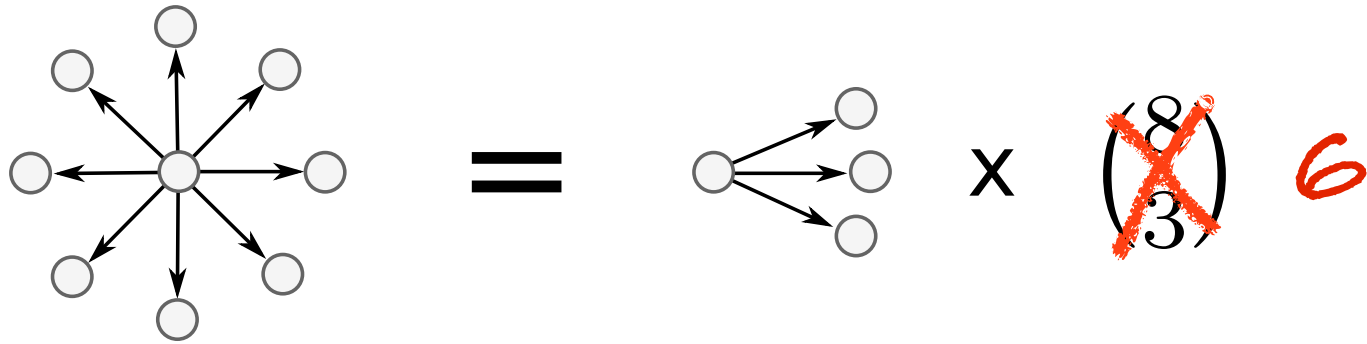


x

~~$\binom{8}{3}$~~

6



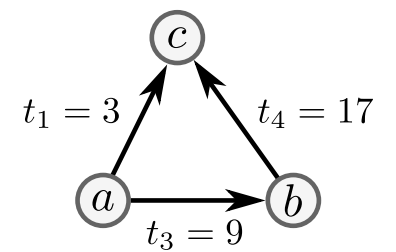
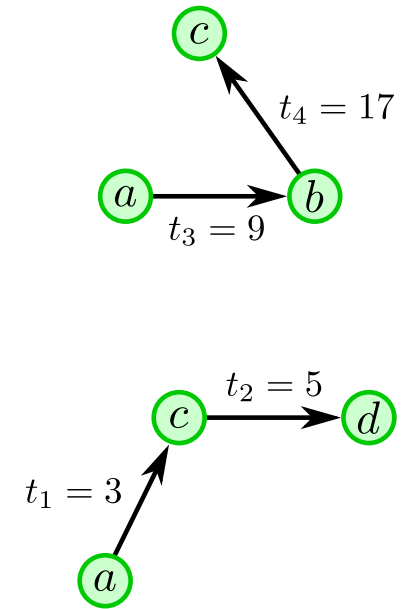
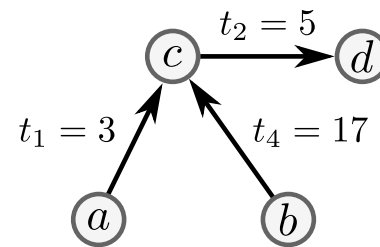
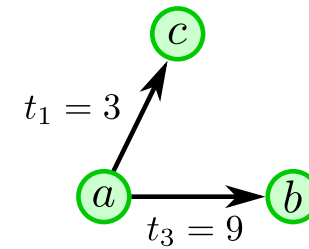
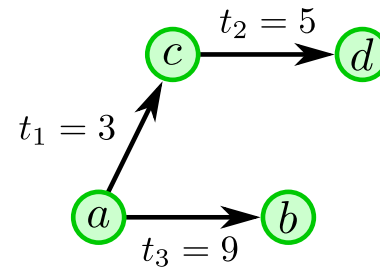
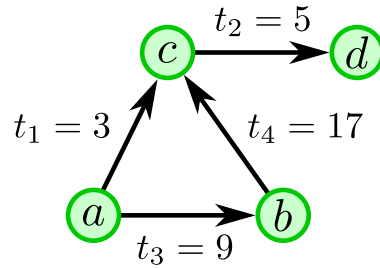
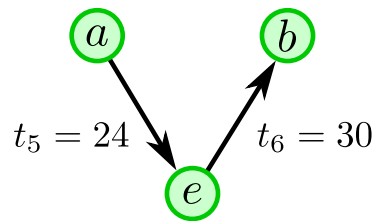
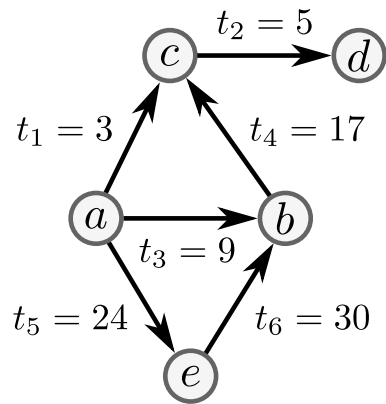


Valid temporal subgraph:

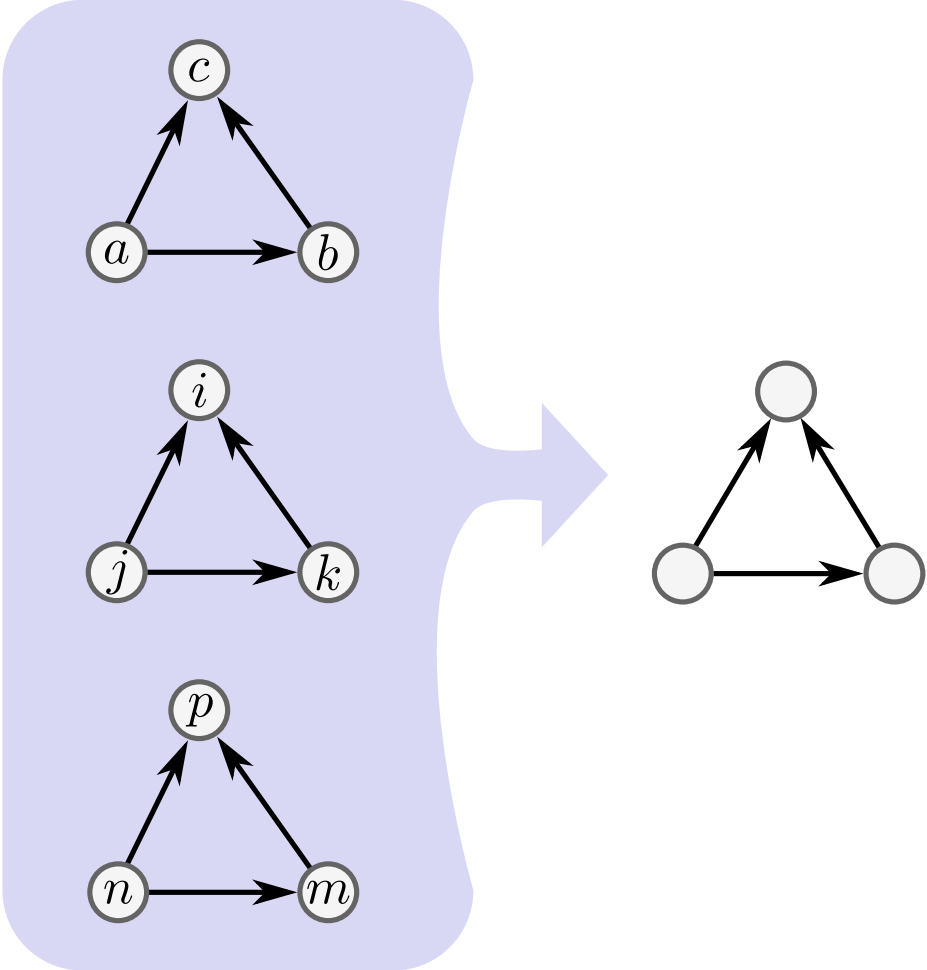
Sets of events where all event pairs are Δt - connected

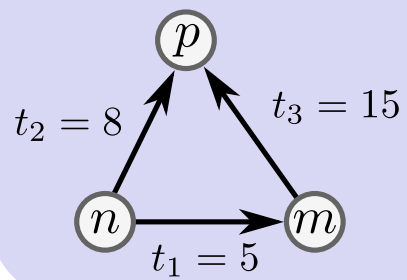
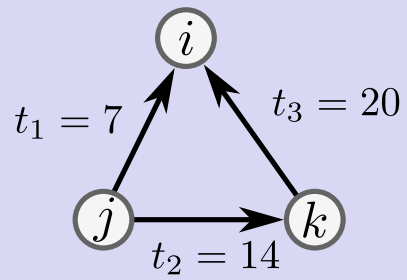
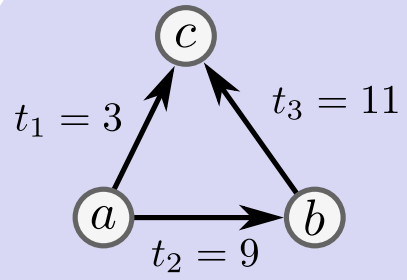
... and all consecutive events of all nodes are included.

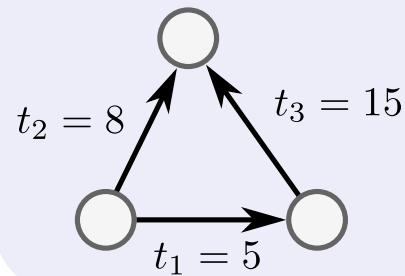
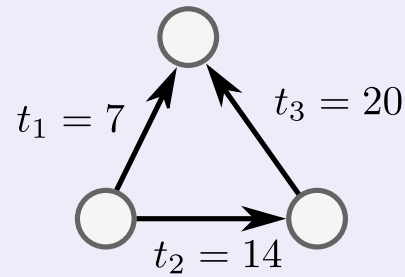
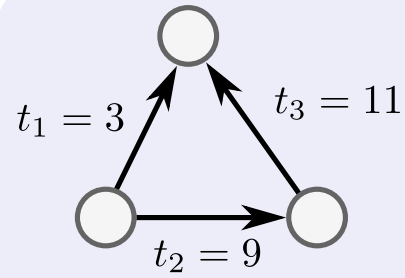
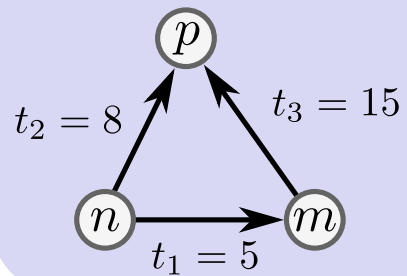
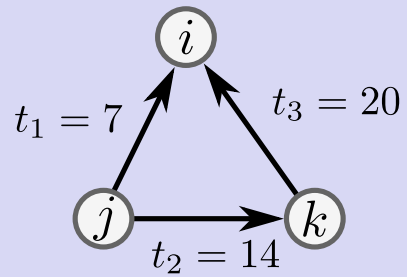
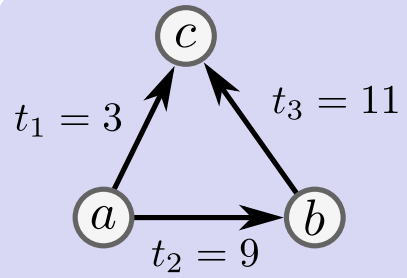
$\Delta t = 10$

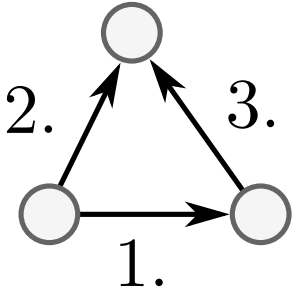
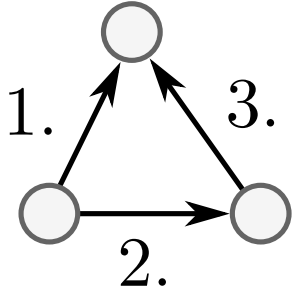
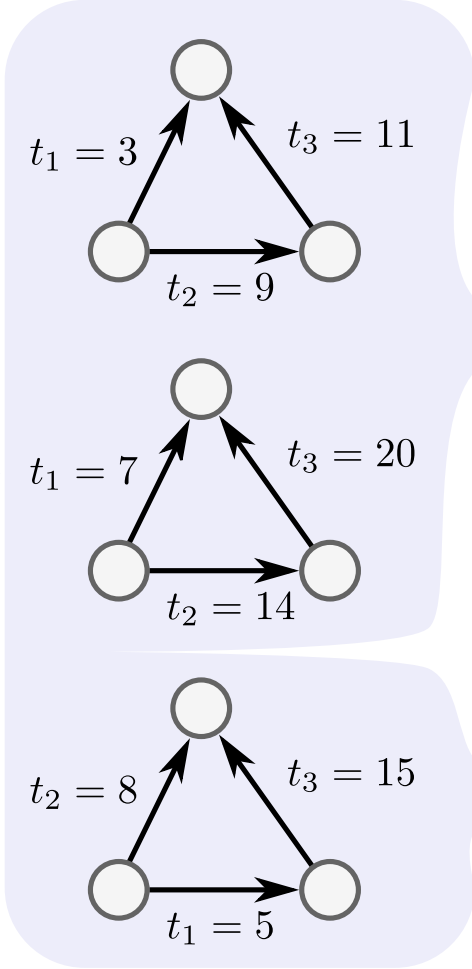
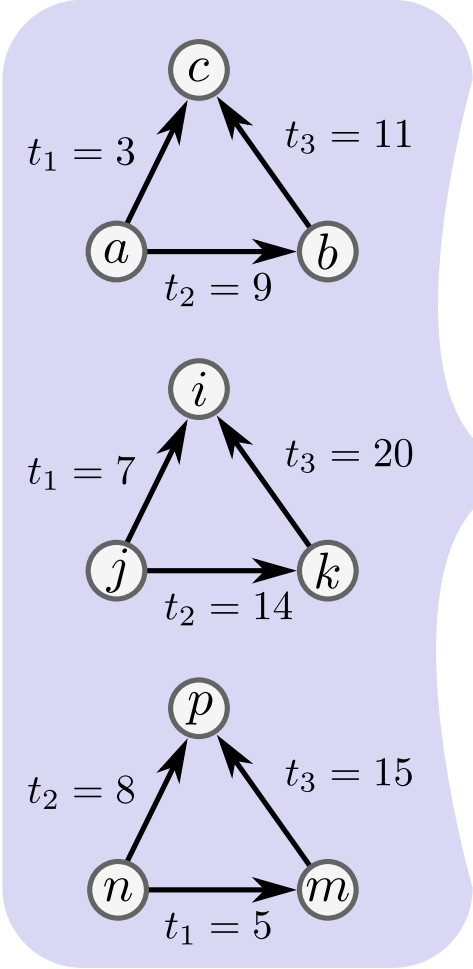


Motif: *Equivalence class* of connected subgraphs.







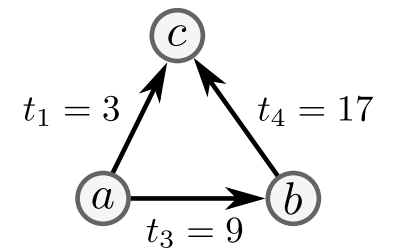
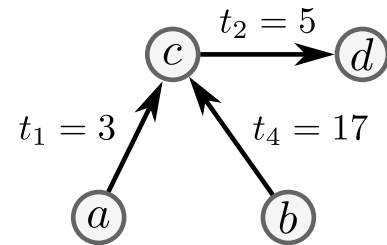
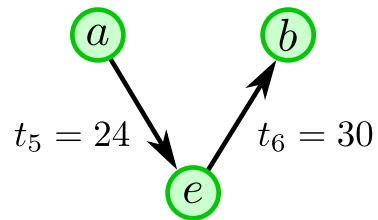
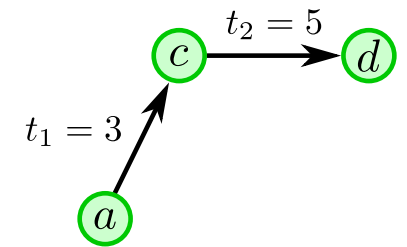
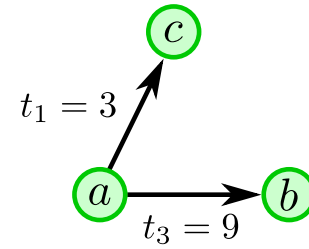
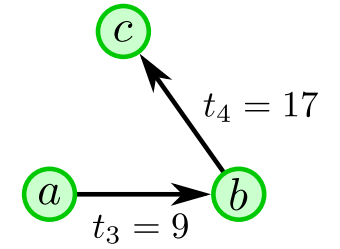
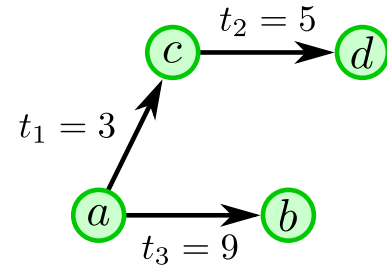
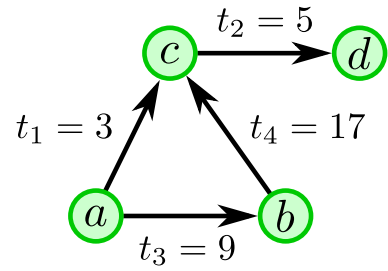
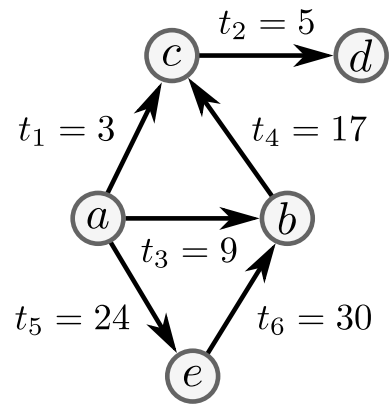


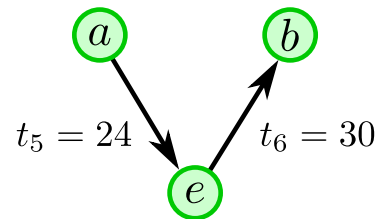
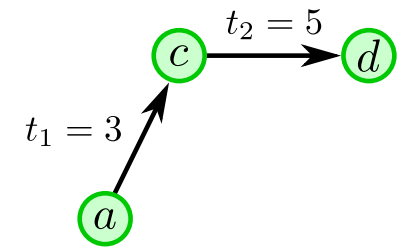
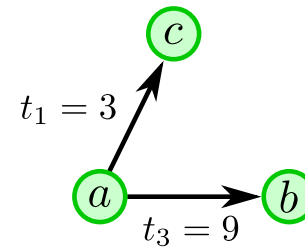
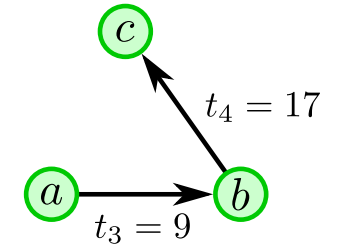
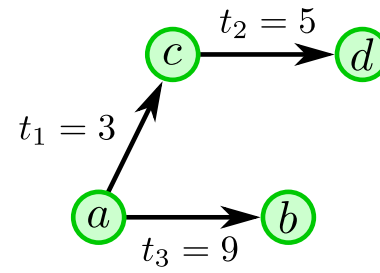
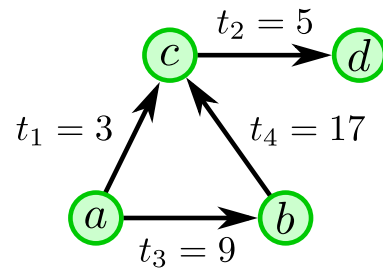
Valid temporal subgraph:

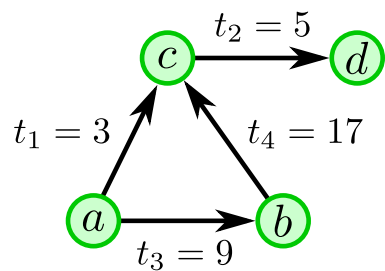
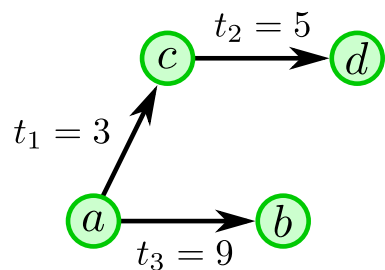
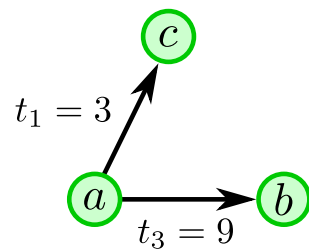
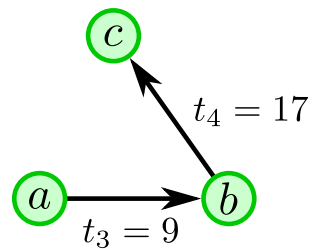
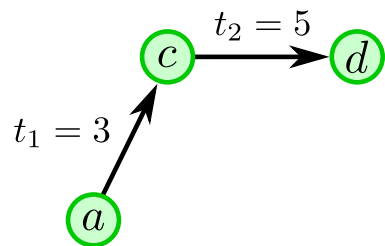
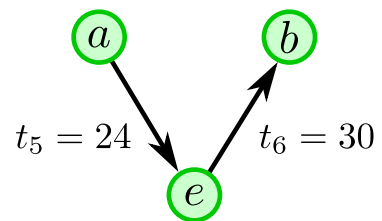
Sets of events where all event pairs are Δt - connected and all consecutive events of all nodes are included.

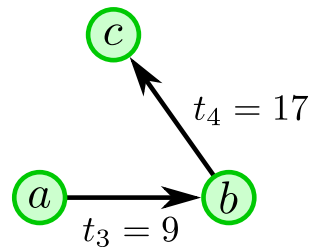
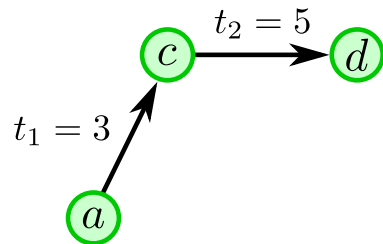
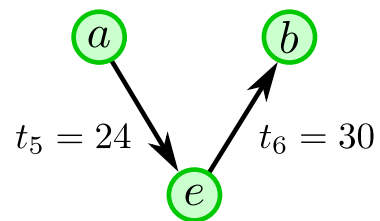
Temporal motif:

Equivalence class of valid temporal subgraphs.

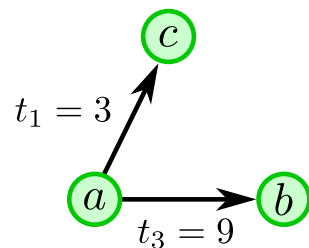
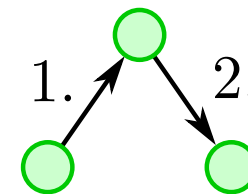




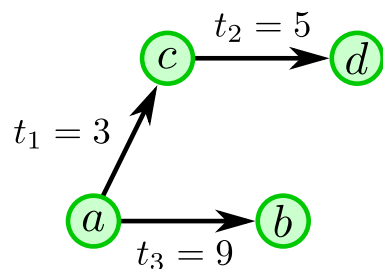
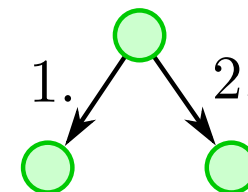




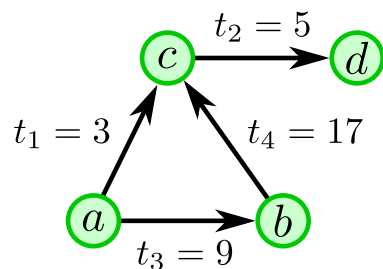
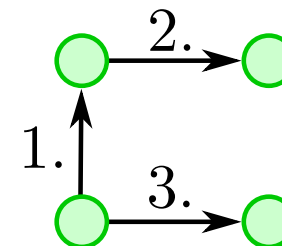
= 3 x



= 1 x



= 1 x



= 1 x

